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# ALPHA STABLE RANDOM FIELDS AND ADDITIVE ERROR

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## ABSTRACT

*This work studies the estimation of spectral density for random field (two-dimensional signal) when the spectral measure have certain mixture and the process is observed with a constant error. The objective of this paper is to give an estimator of the constant error by using the Jackson polynomial kernel. We show that the rate of convergence depends of size of sample and the behaviours of the spectral density at origin. Indeed the estimator converges rapidly when the spectral density is null at origin. Few long memory signals are taken here as example.*

## KEYWORDS

*Spectral density; Jackson kernel; Stable random fields.*

## 1. INTRODUCTION

This work considers the class of symmetric alpha stable signals which are known as signals having infinite energy. These signals have been developed in recent decades by several authors, including [1]-[13], to name a few.

The Gaussian density distribution remains a particular cases of alpha-stable distribution ( $\alpha = 2$ ).

Alpha stable distribution is a better model for signals that are impulsive in nature. It is adapted for signals that their variance is large and the Gaussian can not used for modelling this process. Signals in this class contain high-pitched bursts or occasional spikes.

Symmetric alpha stable signals are used for modeling many phenomenons in several fields: : physics, biology, electronic and electrical engineering, hydrology, economies, communications and radar applications and signal image processing,...see [14]-[25].

In this work, we consider a symmetric  $\alpha$  stable random field  $Z = \{Z_{(n_1, n_2)} : (n_1, n_2) \in Z^2\}$  having the following integral representation:

$$Z_{(n_1, n_2)} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \exp[i(n_1 \lambda_1 + n_2 \lambda_2)] d\xi(\lambda_1, \lambda_2)$$

where  $1 < \alpha < 2$  and  $\xi$  is a complex valued symmetric  $\alpha$ -stable random measure on  $R^2$  with independent and isotropic increments. The measure defined by  $m(AxB) = |\xi(AxB)|_\alpha^\alpha$  (see [4]) is called "control" measure or spectral measure. The case where this measure is absolutely continuous with respect to Lebesgue measure:  $dm(x_1, x_2) = \phi(x_1, x_2) dx_1 dx_2$  is considered in [4], [26], [27]. And the function  $\phi$  called the spectral density of  $Z$  was already estimated by [4],

when the time of the process is continuous, by [26] when the time of the process is discrete and by [27] when the time of the process is p-adic.

This paper considers a case, often encountered in practice namely when we observe this random field with an unknown constant error:  $X_{(n_1, n_2)} = a(n_1, n_2) + Z_{(n_1, n_2)}$ . Thus, the signal observed is  $X_{(n_1, n_2)}$  instead of the signal  $Z_{(n_1, n_2)}$  alone. We also consider a more general case: when the spectral measure is the sum of an absolutely continuous measure with respect to Lebesgue measure, a discrete measure and a finite number of absolutely continuous measures on several lines:

$$dm = \phi(x_1, x_2) dx_1 dx_2 + \sum_{i=1}^q c_i \delta_{(w_{(1,i)}, w_{(2,i)})} + \sum_{i=1}^q \varphi_k \delta_{(u_i, a_k u_i + b_k)}$$

where  $\delta$  is a Dirac measure,  $\phi$  and  $\varphi_k$  are non-negative integrable and bounded functions.  $c_i$  is unknown positive real number and  $w_{(1,i)}, w_{(2,i)}$ ,  $a_k$  and  $b_k$  are unknown real numbers. Assume that  $w_{(1,i)} \neq 0$  and  $w_{(2,i)} \neq 0$ . The estimation of the constant error when the process has one dimension is given in [28]. This mixed measure is encountered when, for example, the resistance of the soil is measured on agricultural land which has a continuous random measurement and when pebbles are randomly encountered the measurement reaches jumps which represent the discrete measure. When the measures are made in places where the passage of tractors is frequent. The rate of convergence will be studied particularly for spectral densities which are zero at the origin as  $\phi(\lambda_1, \lambda_2) = \sin^{2k\alpha} \left( \frac{\lambda_1}{2} \right) \sin^{2k\alpha} \left( \frac{\lambda_2}{2} \right) g(\lambda_1, \lambda_2)$  and  $\phi(\lambda_1, \lambda_2) = |\lambda_1 \lambda_2|^\beta g(\lambda_1, \lambda_2)$ . We show that the convergence speed is much faster depending on the value of the parameter of  $\beta$ .

This paper is organized as follows: The second section gives some properties of Jackson polynomial kernel and an estimator of the constant  $a$ . We show that the estimator converges to  $a$  in probability and converges in  $L_p$  ( $p < \alpha$ ). Since, we assume that the spectral density of  $Z$  is vanishing at origin, we show that the estimator converges more rapidly to  $a$  in accordance with the values of  $\beta$ . The third section consists in illustrating the results found through numerical data. The last section, contains the concluding remarks, the potential applications and the open research problems.

## 2. ESTIMATION OF THE CONSTANT

This paper considers a (S $\alpha$ S) process where its spectral representation is :

$$Z_{(n_1, n_2)} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{i(n_1 \lambda_1 + n_2 \lambda_2)} d\xi(\lambda_1, \lambda_2),$$

where  $\xi$  is an isotropic symmetric  $\alpha$ -stable with independent increments. The measure defined by:  $m([s_1, t_1] \times [s_2, t_2]) = |\xi(s_1, t_1) - \xi(s_2, t_2)|_\alpha^\alpha$  is Lebesgue-Stieltjes measure called *the spectral measure* (see [1], [4]) When  $m$  is absolutely continuous  $d\mu = \phi(x_1, x_2) dx_1 dx_2$ , the function  $\phi$  is called *the spectral density* of the process  $Z$ .

Let  $Z_{(n_1, n_2)}$  observations of the process  $Z$ :  $(Z_{(n_1, n_2)})$  with  $0 \leq n_1 \leq N_1 - 1$  and  $0 \leq n_2 \leq N_2 - 1$ , where  $N_1, N_2$  satisfy:  $N_1 - 1 = 2k(n_1 - 1)$  and  $N_2 - 1 = 2k(n_2 - 1)$  with  $n_1, n_2 \in N$   $k \in N \cup \left\{ \frac{1}{2} \right\}$  if  $k = \frac{1}{2}$  then  $n_1 = 2n'_1 - 1$ ,  $n'_1 \in N$  and  $n_2 = 2n'_2 - 1$ ,  $n'_2 \in N$ .

The Jackson polynomial kernel is defined in [29], [11] and [26], as follows:

$$|H_N(\lambda_1, \lambda_2)|^\alpha = |A_{(N_1, N_2)} H^{(N_1, N_2)}(\lambda_1, \lambda_2)|^\alpha$$

$$\text{where } H^{(N_1, N_2)}(\lambda_1, \lambda_2) = \frac{1}{q_{k, n_1, n_2}} \left( \frac{\sin(\frac{n_1 \lambda_1}{2})}{\sin(\frac{\lambda_1}{2})} \right)^{2k} \left( \frac{\sin(\frac{n_2 \lambda_2}{2})}{\sin(\frac{\lambda_2}{2})} \right)^{2k}$$

$$\text{with } q_{k, n_1, n_2} = \left( \frac{1}{2\pi} \right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left( \frac{\sin(\frac{n_1 \lambda_1}{2})}{\sin(\frac{\lambda_1}{2})} \right)^{2k} \left( \frac{\sin(\frac{n_2 \lambda_2}{2})}{\sin(\frac{\lambda_2}{2})} \right)^{2k} d\lambda_1 d\lambda_2$$

In addition, we have  $A_{(N_1, N_2)} = (B_{\alpha, N_1, N_2})^{-\frac{1}{\alpha}}$  with  $B_{\alpha, N_1, N_2} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |H^{(N_1, N_2)}(\lambda_1, \lambda_2)|^\alpha d\lambda_1 d\lambda_2$ .

The estimator:

In this paper, we construct the following estimate of the error  $a$ :

$$\hat{a} = \frac{A_{(N_1, N_2)}}{H_N(0, 0)} \sum_{n'=-k(n_1-1)}^{k(n_1-1)} \sum_{n''=-k(n_2-1)}^{k(n_2-1)} X(n' + k(n_1 - 1), n'' + k(n_2 - 1)). \quad (1)$$

We start by showing that the estimator  $\hat{a}$  converges to  $a$  in probability and converges in  $L_p$  ( $p < \alpha$ ). Then we show that these convergences are faster for signals whose spectral density vanishes at the origin. This means that the estimator is slowed down by the disturbance of the energy at the origin. To lighten the formulas and make the paper easy to read, we delete in the rest of this paper the last term of the expression of the spectral measure. Thus, henceforth the considered spectral measure becomes:

$$dm = \phi(x_1, x_2) dx_1 dx_2 + \sum_{i=1}^q c_i \delta_{(w_{(1,i)}, w_{(2,i)})}$$

Citing now two lemmas given properties of Jackson polynomial kernel that we will use later. Their proofs are given [26], [29].

### Lemma 1

The function  $H^{(N_1, N_2)}(\lambda_1, \lambda_2)$  can be written as follows:

$$H^{(N_1, N_2)}(\lambda_1, \lambda_2) = \sum_{m_1=-k(n_1-1)}^{k(n_1-1)} \sum_{m_2=-k(n_2-1)}^{k(n_2-1)} h_k\left(\frac{m_1}{n_1}\right) \cos(m_1 \lambda_1) h_k\left(\frac{m_2}{n_2}\right) \cos(m_2 \lambda_2),$$

where  $h_k$  is a even positive function.

### Lemma 2.

Let

$$B'_{\alpha, N_1, N_2} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| \frac{\sin \frac{n_1 \lambda_1}{2}}{\sin \frac{\lambda_1}{2}} \right|^{2k\alpha} \left| \frac{\sin \frac{n_2 \lambda_2}{2}}{\sin \frac{\lambda_2}{2}} \right|^{2k\alpha} d\lambda_1 d\lambda_2$$

and

$$J_{N_1, N_2, \alpha} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |(u_1, u_2)|^\gamma |H_{N_1, N_2}(\lambda_1, \lambda_2)|^\alpha d\lambda_1 d\lambda_2,$$

where  $\gamma \in ]0, 2]$ . Then we have the following inequalities,

$$B'_{\alpha, N_1, N_2} \begin{cases} \geq \left(2\pi \left(\frac{2}{\pi}\right)^{2k\alpha}\right)^2 n_1^{2k\alpha-1} n_2^{2k\alpha-1} & \text{if } 0 < \alpha < 2 \\ \leq \left(\frac{4\pi k\alpha}{2k\alpha-1}\right)^2 n_1^{2k\alpha-1} n_2^{2k\alpha-1} & \text{if } \frac{1}{2k} < \alpha < 2 \end{cases}$$

and

$$J_{N_1, N_2, \alpha} \leq \begin{cases} \left(\frac{\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma-2k\alpha)}\right)^2 \frac{1}{n_1^{2k\alpha-1}} \frac{1}{n_2^{2k\alpha-1}} & \text{if } \frac{1}{2k} < \alpha < \frac{\gamma+1}{2k}, \\ \left(\frac{2k\alpha\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma+1)(2k\alpha-\gamma-1)}\right)^2 \frac{1}{(n_1 n_2)^\gamma} & \text{if } \frac{\gamma+1}{2k} < \alpha < 2. \end{cases}$$

### Theorem 1

Let  $p$  a real number such that  $0 < p < \alpha$ . We have

$$|\hat{a} - a|^p = o\left(\frac{1}{(n_1 n_2)^{\frac{p}{\alpha}}}\right)$$

**Proof** Using the spectral representation of the process, we obtain

$$\hat{a} = \frac{A_{(N_1, N_2)}}{H_{(N_1, N_2)}(0, 0)} \sum_{n'=-k(n_1-1)}^{k(n_1-1)} \sum_{n''=-k(n_2-1)}^{k(n_2-1)} h_k\left(\frac{n'}{n_1}, \frac{n''}{n_2}\right) \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \exp[i([n' + k(n_1 - 1)]\lambda_1) + i([n'' + k(n_2 - 1)]\lambda_2)] d\xi(\lambda_1, \lambda_2) + a$$

From [1], we can write the characteristic function of  $(\hat{a} - a)$ :

$$E \exp[i\Re r \bar{r}(\hat{a} - a)] = \exp - C_\alpha |r|^\alpha \int_{-\pi}^{\pi} \left| \frac{A_{(N_1, N_2)}}{H_{(N_1, N_2)}(0, 0)} \sum_{n'=-k(n_1-1)}^{k(n_1-1)} \sum_{n''=-k(n_2-1)}^{k(n_2-1)} h_k\left(\frac{n'}{n_1}, \frac{n''}{n_2}\right) e^{in'\lambda_1} e^{in''\lambda_2} \right|^\alpha d\xi(\lambda_1, \lambda_2)$$

where  $r = r_1 + ir_2$ . It is easy to show that:

$E \exp[i\Re r \bar{r}(\hat{a} - a)] = \exp(-C_\alpha |r|^\alpha \psi_{(N_1, N_2)})$ , where  $\psi_{(N_1, N_2)} = \psi_{N_1, N_2, 1} + \psi_{N_1, N_2, 2}$  with

$$\psi_{N_1, N_2, 1} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{|H_{(N_1, N_2)}(\lambda_1, \lambda_2)|^\alpha}{|H_{(N_1, N_2)}(0, 0)|^\alpha} \phi(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \quad \text{and}$$

$$\psi_{N_1, N_2, 2} = \sum_{i=1}^q c_i \frac{|H_{(N_1, N_2)}(w_{i,1}, w_{i,2})|^\alpha}{|H_{(N_1, N_2)}(0,0)|^\alpha}.$$

The function  $\phi$  being bounded on  $[-\pi, \pi]^2$  and  $|H_{(N_1, N_2)}(\cdot, \cdot)|^\alpha$  being a kernel, it can be shown that  $\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |H_{(N_1, N_2)}(\lambda_1, \lambda_2)|^\alpha \phi(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$  is converging to  $\phi(0,0)$ . On the other hand, from lemma 2, we have:

$$\frac{1}{|H_{(N_1, N_2)}(0,0)|^\alpha} = \frac{B'_{\alpha, N_1, N_2}}{n_1^{2k\alpha} n_2^{2k\alpha}} = O\left(\frac{1}{n_1 n_2}\right) \quad (2)$$

Therefore  $\psi_{N_1, N_2, 1}$  converges to 0.

$$\psi_{N_1, N_2, 2} \leq \sum_{i=1}^q \frac{c_i}{B'_{\alpha, N_1, N_2}} \frac{1}{\left| \sin\left[\frac{w_{i,1}}{2}\right] \sin\left[\frac{w_{i,2}}{2}\right] \right|^{2k\alpha}} \frac{B'_{\alpha, N_1, N_2}}{n_1^{2k\alpha} n_2^{2k\alpha}}.$$

Therefore  $\psi_{N_1, N_2, 2} = O\left(\frac{1}{n_1^{2k\alpha} n_2^{2k\alpha}}\right)$ . Thus

$$\psi_{(N_1, N_2)} = O\left(\frac{1}{n_1 n_2}\right).$$

Consequently, the characteristic function of  $\hat{a} - a$  converges to 1 when  $N_1$  and  $N_2$  tend to infinity. We deduce that  $\hat{a}$  converges to  $a$  in probability.

In order to study the convergence in  $L_p$  where  $0 < p < \alpha$ , we put

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - e^{ir \cos \theta}}{|r|^{1+p}} dr d\theta.$$

Assuming now  $r = \varepsilon r'$ ,  $\theta = \tau' - \tau_0$

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4} + \tau_0}^{\frac{\pi}{4} + \tau_0} \frac{1 - e^{i\varepsilon r' \cos(\tau' - \tau_0)}}{|\varepsilon|^{1+p} |r'|^{1+p}} \varepsilon dr' \tau'$$

$$D_p |x|^p = \Re e \int_0^{\infty} \int_{-\frac{\pi}{4} + \tau_0}^{\frac{\pi}{4} + \tau_0} \frac{1 - e^{i\Re e(\bar{t}x)}}{t^{1+p}} d|t| d\theta' - \Re e \int_{-\infty}^0 \int_{-\frac{\pi}{4} + \tau_0}^{\frac{\pi}{4} + \tau_0} \frac{1 - e^{i\Re e(\bar{t}x)}}{|t|^{1+p}} d|t| d\theta'.$$

Let us substitute  $x$  by  $\hat{a} - a$ , we have

$$D_p E|\hat{a} - a|^p = \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4} + \tau_0}^{\frac{\pi}{4} + \tau_0} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_{(N_1, N_2)}}}{|t|^{1+p}} d|t| d\theta' = \frac{\pi}{2} \int_{-\infty}^{\infty} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} dt.$$

Let  $u = t[\psi_{(N_1, N_2)}]^{1/\alpha}$  and using (2), we obtain

$$\frac{2}{\pi} C_{p, \alpha} E|\hat{a} - a|^p = (\psi_{(N_1, N_2)})^{\frac{p}{\alpha}} = O\left(\frac{1}{n_1^{p/\alpha} n_2^{p/\alpha}}\right). \quad (3)$$

where  $C_{p,\alpha} = R_p F_{p,\alpha}^{-1}(C_\alpha)^{-\frac{p}{\alpha}}$

with  $R_p = \int \frac{1-\cos(u)}{|u|^{1+p}} du$  and  $F_{p,\alpha} = \int \frac{1-e^{-|u|^\alpha}}{|u|^{\frac{1+p}{\alpha}}} du$ .

### Improvement of the rate of convergence

We take the case where the spectral density vanishes at the origin. The following theorems will show that the speed of convergence is better.

#### **Theorem 2**

Assume that the spectral density is satisfying:

$$\phi(\lambda_1, \lambda_2) = |\lambda_1|^\beta |\lambda_2|^\beta g(\lambda_1, \lambda_2)$$

where  $\beta \in ]0, 2k\alpha - 1[$ ,  $\lambda_1, \lambda_2 \in [-\pi, \pi]$  and  $g(\lambda_1, \lambda_2)$  is a bounded function on  $[-\pi, \pi]^2$ , continuous in neighborhood of  $(0,0)$  and  $g(0,0) \neq 0$ . Then

$$2^{8kp} L \leq \lim_{N_1, N_2 \rightarrow \infty} (n_1 n_2)^{\frac{p(\beta+1)}{\alpha}} E|\hat{a} - a|^p \leq \pi^{8kp} L,$$

where  $L$  is the following constant:

$$L = \frac{\pi}{2C_{p,\alpha}} \left[ g(0,0) \int_{-\infty}^{\infty} \frac{|\sin \frac{v}{2}|^{2k\alpha}}{|v|^{2k\alpha-\beta}} dv \right]^{\frac{2p}{\alpha}}.$$

#### **Proof:**

From (2), the function  $\psi_{(N_1, N_2)}$  can be written as:

$$\begin{aligned} \psi_{(N_1, N_2)} = (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| \frac{\sin \frac{n_1 \lambda_1}{2}}{\sin \frac{\lambda_1}{2}} \right|^{2k\alpha} \left| \frac{\sin \frac{n_2 \lambda_2}{2}}{\sin \frac{\lambda_2}{2}} \right|^{2k\alpha} |\lambda_1 \lambda_2|^\beta g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \\ + (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right]} \frac{\sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha} \end{aligned}$$

Using the following inequality:

$$|\sin x| \geq x\pi \quad 0 \leq x \leq \pi, \quad (4)$$

we maximize  $\psi_{(N_1, N_2)}$  as follows:

$$\psi_{(N_1, N_2)} \leq (\pi^{4k\alpha})^2 (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{|\sin \frac{n_1 \lambda_1}{2}|^{2k\alpha}}{|\lambda_1|^{2k\alpha-\beta}} \frac{|\sin \frac{n_2 \lambda_2}{2}|^{2k\alpha}}{|\lambda_2|^{2k\alpha-\beta}} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$$

$$+(n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{1}{\sin\left[\frac{w_{i,1}}{2}\right]} \frac{1}{\sin\left[\frac{w_{i,2}}{2}\right]} \right|^{2k\alpha}.$$

Putting  $n_1 \lambda_1 = u_1$  and  $n_2 \lambda_2 = u_2$ , we have

$$\begin{aligned} \psi_{(N_1, N_2)} &\leq (\pi^{4k\alpha})^2 (n_1 n_2)^{-1-\beta} \int_{-\infty}^{\infty} \frac{|\sin \frac{u_1}{2}|^{2k\alpha}}{|u_1|^{2k\alpha-\beta}} \frac{|\sin \frac{u_2}{2}|^{2k\alpha}}{|u_2|^{2k\alpha-\beta}} g\left(\frac{u_1}{n_1}, \frac{u_2}{n_2}\right) du_1 du_2 \\ &\quad + \frac{(n_1 n_2)^{-2k\alpha+1+\beta}}{(\pi^{4k\alpha})^2} \sum_{i=1}^q c_i \left| \frac{1}{\sin\left[\frac{w_{i,1}}{2}\right]} \frac{1}{\sin\left[\frac{w_{i,2}}{2}\right]} \right|^{2k\alpha}. \end{aligned}$$

From Lemma 2, and the theorem of Lebesgue's dominated convergence we obtain that:

$$\lim_{N_1, N_2 \rightarrow \infty} (n_1 n_2)^{p(\beta+1)\alpha} (\psi_{(N_1, N_2)})^{p\alpha} \leq \left( \pi^{4kp} \left( g(0,0) \int_{-\infty}^{+\infty} |\sin u|^{2k\alpha} |u|^{2k\alpha-\beta} du \right)^{p\alpha} \right)^2.$$

Thus  $\psi_{(N_1, N_2)}$  converges to zero. Using the following inequality

$$|\sin x| \leq |x| \quad \forall x \in [-\pi, \pi], \quad (5)$$

we obtain:

$$\begin{aligned} \psi_{(N_1, N_2)} &\geq 2^{8k\alpha} (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{|\sin \frac{n_1 \lambda_1}{2}|^{2k\alpha}}{|\lambda_1|^{2k\alpha-\beta}} \frac{|\sin \frac{n_2 \lambda_2}{2}|^{2k\alpha}}{|\lambda_2|^{2k\alpha-\beta}} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \\ &\quad + (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin\left[\frac{n_1 w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]} \frac{\sin\left[\frac{n_2 w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]} \right|^{2k\alpha} \\ \psi_{(N_1, N_2)} &\geq 2^{8k\alpha} (n_1 n_2)^{-\beta-1} \left[ \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{|\sin \frac{u_1}{2}|^{2k\alpha}}{|u_1|^{2k\alpha-\beta}} \frac{|\sin \frac{u_2}{2}|^{2k\alpha}}{|u_2|^{2k\alpha-\beta}} g\left(\frac{u_1}{n_1}, \frac{u_2}{n_2}\right) du_1 du_2 + R_n \right], \end{aligned}$$

$$\text{where } R_n = \frac{(n_1 n_2)^{-2k\alpha+\beta+1}}{2^{8k\alpha}} \sum_{i=1}^q c_i \left| \frac{\sin\left[\frac{n_1 w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]} \frac{\sin\left[\frac{n_2 w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]} \right|^{2k\alpha}.$$

Since  $R_n$  converges to zero, the equality (5) gives

$$\lim_{N_1, N_2 \rightarrow \infty} (n_1 n_2)^{\frac{p(\beta+1)}{\alpha}} (\psi_{(N_1, N_2)})^{\frac{p}{\alpha}} \geq 2^{8kp} \left( g(0,0) \int_{-\infty}^{+\infty} \left| \sin\left(\frac{u}{2}\right) \right|^{2k\alpha} / |u|^{2k\alpha-\beta} du \right)^{2p/\alpha}.$$

We arrive at the result of this theorem and this because of equality (3).

### Theorem 3

Assuming that the spectral density satisfies :

$$\phi(\lambda_1, \lambda_2) = \left( \sin \frac{n_2 \lambda_2}{2} \sin \frac{n_1 \lambda_1}{2} \right)^{2k\alpha} g(\lambda_1, \lambda_2) \text{ where } g \text{ is real function, integrable on } [-\pi, \pi] \text{ and}$$

$g(0,0) \neq 0$ . Then

$$\text{cte} \left( \frac{\pi}{2C_{p,\alpha}} \right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \leq \lim_{N_1, N_2 \rightarrow \infty} (n_1 n_2)^{2kp} E|\hat{a} - a|^p \leq$$

$$\left( \frac{\pi}{2C_{p,\alpha}} \right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 + \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right] \sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right] \sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha}$$

**Proof:** From the definition of  $\psi_{(N_1, N_2)}$  and (2), we have

$$\psi_{(N_1, N_2)} = (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left| \sin \frac{n_1 \lambda_1}{2} \sin \frac{n_2 \lambda_2}{2} \right|^{2k\alpha} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$$

$$+ (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right] \sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right] \sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha}.$$

$$\psi_{(N_1, N_2)} \leq (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$$

$$+ (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right] \sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right] \sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha}$$

The lemma 2:

$$\lim_{N_1, N_2 \rightarrow \infty} \psi_{(N_1, N_2)} (n_1 n_2)^{2k\alpha} \leq \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 + \sum_{i=1}^q c_i \left| \frac{1}{\sin \left[ \frac{w_{i,1}}{2} \right]} \frac{1}{\sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha}.$$

To minimize  $\psi_{(N_1, N_2)}$ , using the properties of sinus we obtain

$$\psi_{(N_1, N_2)} \geq (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ \left( \sin \frac{n_1 \lambda_1}{2} \sin \frac{n_2 \lambda_2}{2} \right)^2 \right]^{[k\alpha]+1} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$$

$$+ (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right] \sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right] \sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha}$$

where  $[k\alpha]$  represents the integer part of  $k\alpha$ , we obtain

$$\psi_{(N_1, N_2)} \geq (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$$

$$+ (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right] \sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right] \sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha}$$

Using the binomial formula we have:

$$\left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} =$$

$$\sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \left( \cos^r \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} \cos^{(2[k\alpha]+2-r)} \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right)$$

Using again the binomial formula, we obtain:

$$\cos^r(a) = (e^a + e^{-ia}2)^r = 12^r \sum_{j=0}^r C_r^j e^{ija} e^{-i(r-j)a}.$$

Hence

$$\begin{aligned} & \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} = \\ & \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \left( \frac{1}{2^r} \sum_{j=0}^r C_r^j e^{ij \frac{n_1 \lambda_1 - n_2 \lambda_2}{2}} e^{-i(r-j) \frac{n_1 \lambda_1 + n_2 \lambda_2}{2}} \right) \\ & \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} = \\ & \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r e^{-i(r) \frac{n_2 \lambda_2}{2}} \left( 12^r \sum_{j=0}^r C_r^j e^{i(2jr) \frac{n_1 \lambda_1}{2}} \right) \\ & \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 = \\ & \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \int_{-\pi}^{\pi} e^{-i(r) \frac{n_2 \lambda_2}{2}} \left( \frac{1}{2^r} \sum_{j=0}^r C_r^j \int_{-\pi}^{\pi} e^{i(2jr) \frac{n_1 \lambda_1}{2}} g(\lambda_1, \lambda_2) d\lambda_1 \right) d\lambda_2 \end{aligned}$$

The function  $g(\cdot, \lambda_2)$  being even for all  $\lambda_2 \in [-\pi, \pi]$ , we have

$$\begin{aligned} & \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 = \\ & \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \int_{-\pi}^{\pi} e^{-i(r) \frac{n_2 \lambda_2}{2}} \left( \frac{1}{2^r} \sum_{j=0}^r C_r^j \int_{-\pi}^{\pi} \cos \left( (2jr) \frac{n_1 \lambda_1}{2} \right) g(\lambda_1, \lambda_2) d\lambda_1 \right) d\lambda_2 \end{aligned}$$

The function  $g(\lambda_1, \cdot)$  being even for all  $\lambda_1 \in [-\pi, \pi]$ , we have

$$\begin{aligned} & \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 = \\ & \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \int_{-\pi}^{\pi} \cos \left( -ir \frac{n_2 \lambda_2}{2} \right) \left( \frac{1}{2^r} \sum_{j=0}^r C_r^j \int_{-\pi}^{\pi} \cos \left( (2jr) \frac{n_1 \lambda_1}{2} \right) g(\lambda_1, \lambda_2) d\lambda_1 \right) d\lambda_2 \end{aligned}$$

Since  $\frac{1}{2^r} \sum_{j=0}^r C_r^j \int_{-\pi}^{\pi} \cos \left( (2jr) \frac{n_1 \lambda_1}{2} \right) g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$  converges to  $\int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$  uniformly in  $\lambda_2$  when  $r$  is even and converges to 0 when  $r$  is odd. Therefore

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left[ \cos \frac{n_1 \lambda_1 - n_2 \lambda_2}{2} - \cos \frac{n_1 \lambda_1 + n_2 \lambda_2}{2} \right]^{2[k\alpha]+2} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$$

Converges to  $\sum_{p=1}^{\frac{2[k\alpha]+2}{2}} C_{2[k\alpha]+2}^{2p} 12^{2p} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$ .

Since  $\lim_{N_1, N_2 \rightarrow \infty} (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left| \frac{\sin \left[ \frac{n_1 w_{i,1}}{2} \right]}{\sin \left[ \frac{w_{i,1}}{2} \right]} \frac{\sin \left[ \frac{n_2 w_{i,2}}{2} \right]}{\sin \left[ \frac{w_{i,2}}{2} \right]} \right|^{2k\alpha} = 0$ . The similar arguments are used for showing that

$$\lim_{N_1, N_2 \rightarrow \infty} (\psi_{(N_1, N_2)})^{\frac{p}{\alpha}} (n_1 n_2)^{2kp} \geq cte \left( \int_{-\pi}^{\pi} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \right)^{p\alpha}. \quad (6)$$

**Theorem 4**

Assume that spectral density satisfies:

$$\phi(\lambda_1\lambda_2) = |\lambda_1|^\beta |\lambda_2|^\beta g(\lambda_1, \lambda_2)$$

where  $g$  is real positive function and  $\beta > 2k\alpha - 1$ . Suppose that  $g$  is bounded on  $[-\pi, \pi]^2$ . Then

$$cte \times R \leq \lim_{N_1, N_2 \rightarrow \infty} (n_1 n_2)^{2kp} E|\hat{a} - a|^p \leq R$$

$$\text{where } R = \left(\frac{\pi}{2C_{p,\alpha}}\right)^2 \left(\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left(\frac{|\lambda_1|^\beta}{|\sin \frac{\lambda_1}{2}|^{2k\alpha}} \frac{|\lambda_2|^\beta}{|\sin \frac{\lambda_2}{2}|^{2k\alpha}} g(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2\right)^{p\alpha}.\right.$$

**Proof:**

Define the continuous function  $l$  as follows:

$$l(\lambda) = \begin{cases} \pi^{2k\alpha} & \text{if } |\lambda| > \pi \\ |\lambda|^{2k\alpha} \left|\sin \frac{\lambda}{2}\right|^{2k} & \text{if } 0 < |\lambda| \leq \pi \\ 2^{2k\alpha} & \text{if } \lambda = 0. \end{cases}$$

The function  $\psi_{(N_1, N_2)}$  can be written as:

$$\begin{aligned} \psi_{(N_1, N_2)} = & (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left|\frac{\sin \frac{n_1 \lambda_1}{2}}{\sin \frac{\lambda_1}{2}}\right|^{2k\alpha} \sin^{2k\alpha} \left(\frac{\lambda_1}{2}\right) \left|\frac{\sin \frac{n_2 \lambda_2}{2}}{\sin \frac{\lambda_2}{2}}\right|^{2k\alpha} \sin^{2k\alpha} \left(\frac{\lambda_2}{2}\right) h(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 + \\ & (n_1 n_2)^{-2k\alpha} \sum_{i=1}^q c_i \left|\frac{\sin \left[\frac{n_1 w_{i,1}}{2}\right] \sin \left[\frac{n_1 w_{i,2}}{2}\right]}{\sin \left[\frac{w_{i,1}}{2}\right] \sin \left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha} \end{aligned}$$

where  $h(\lambda_1, \lambda_2) = l(\lambda_1)l(\lambda_2)|\lambda_1\lambda_2|^{\beta-2k\alpha} g(\lambda_1, \lambda_2)$ . The function  $h$  is integrable on  $[-\pi, \pi]^2$ . Thus

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} h(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \leq \sup(g) \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} l(\lambda_1)l(\lambda_2)|\lambda_1\lambda_2|^{\beta-2k\alpha} d\lambda_1 d\lambda_2.$$

Using the inequality (6), we obtain  $l(\lambda_1) \leq \pi^{2k\alpha}$  and  $l(\lambda_2) \leq \pi^{2k\alpha}$ . Thus

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} h(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \leq 4\pi^{4k\alpha} \sup(g) \int_0^{\pi} \int_0^{\pi} (\lambda_1 \lambda_2)^{\beta-2k\alpha} d\lambda_1 d\lambda_2.$$

Since  $\beta > 2k\alpha - 1$ , the function  $h$  is integrable. From (3) and the theorem 1, the result is obtained.

### 3. NUMERICAL STUDIES

We give the simulation of the studied process:

$$Z_{n_1, n_2} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{i(n_1 \lambda_1 + n_2 \lambda_2)} d\xi(\lambda_1, \lambda_2), \quad (7)$$

where  $1 < \alpha < 2$  and  $\xi$  is a complex symmetric  $\alpha$ -stable measure on  $R$  with independent and isotropic increments and with control measure  $m$  such that  $dm = \phi(x_1, x_2) dx_1 dx_2$ .

For that, we use the series representations given by [30]). Therein the authors have shown that the process  $Z$  defined by (8) can be expressed as follows:

$$Z_{n_1, n_2} = C_\alpha \left( \int \phi(x_1, x_2) dx_1 dx_2 \right)^{1/\alpha} \sum_{k=1}^{\infty} \varepsilon_k \Gamma_k^{-1/\alpha} e^{i(n_1 V_{1,k} + n_2 V_{2,k})} e^{i\theta_k} \quad (8)$$

where

- $\varepsilon_k$  is a sequence of i.i.d. random variables such as  $P[\varepsilon_k = 0] = P[\varepsilon_k = 1] = \frac{1}{2}$ ,
- $\Gamma_k$  is a sequence of arrival times of Poisson process,
- $(V_{1,k}, V_{2,k})$  is a couple of sequence of i.i.d. random variables independent of  $\varepsilon_k$  and of  $\Gamma_k$  having the same joint distribution of control measure  $m$ , which has probability density  $\phi$
- $\theta_k$  is a sequence of i.i.d. random variables that have the uniform distribution on  $[-\pi, \pi]$ , independent of  $\varepsilon_k, V_{1,k}, V_{2,k}$  and  $\Gamma_k$ .

For the simulation of  $N_1$  and  $N_2$  values of the process  $Z_{n_1, n_2}$  where

$(N_1, N_2 = 101, 501, 1001, 1501, 2001)$ , we generate:

- 2000 values of  $\varepsilon_k$
- 2000 values of  $\Gamma_k$
- 2000 values of  $V_k$
- 2000 values of  $\theta_k$

and calculate for all  $0 \leq n_1 \leq N_1$  and  $0 \leq n_2 \leq N_2$

$$Z_{n_1, n_2} = C_\alpha \left( \int \phi(x_1, x_2) dx_1 dx_2 \right)^{1/\alpha} \sum_{k=1}^{2000} \varepsilon_k \Gamma_k^{-1/\alpha} e^{i(n_1 V_{1,k} + n_2 V_{2,k})} e^{i\theta_k}$$

The spectral density is taken as  $\phi(x_1, x_2) = |x_1 x_2|^\beta e^{-|x_1| - |x_2|}$  for  $x_1, x_2 \in [-\pi, \pi]$  and  $\phi(x_1, x_2) = 0$  otherwise and  $\alpha = 1,68$  and  $k = 4$ . The value of  $\beta$  is taken so as to have two cases:  $\beta$  greater than  $2k\alpha - 1$  and  $\beta$  less than  $2k\alpha - 1$ . Afterwards, we generate :

$X_{n_1, n_2} = a + Z_{n_1, n_2}$  where  $a$  is chosen equal to 45.

We calculate the estimator  $\hat{a}$  given in (1) for different sizes of sample  $N_1, N_2 = 101, 501, 1001, 1501, 2001$ . The result is given in the following table:

$2k\alpha - 1 = 12.44$	$\beta = 0.5$	$\beta = 30$
N1= N2=101	$\hat{a} = 30.4$	$\hat{a} = 37$
N1= N2=501	$\hat{a} = 34.7$	$\hat{a} = 48$
N1= N2=1001	$\hat{a} = 50.2$	$\hat{a} = 42.2$
N1= N2=1501	$\hat{a} = 39.9$	$\hat{a} = 47.1$
N1= N2=1201	$\hat{a} = 43.8$	$\hat{a} = 44.2$

We find that the estimator converges more and more to  $a$  ( $a=45$ ) when the size of the sample increases. The approximation is more accurate when  $\beta$  is larger than  $2k\alpha - 1$  compared to the case where  $\beta$  is smaller than  $2k\alpha - 1$ .

#### 4. CONCLUSIONS

This work presents an estimate of the constant error, which is added to the observation of the signal (SaS) when the spectral measurement is the sum of a continuous part and a few jumps (mixed measurement). The paper shows that the absence of the energy of signal at frequencies close to zero allows rapid convergence of the estimator. In other words, the energy at frequencies close to zero slows down and disturbs the convergence of our estimator.

This work may have several applications where processes have high variability and are disturbed by constant noise. For instance:

- Dynamic images of an agricultural field taken by drones, to identify weeds. These images can have a great variance, in particular when the images are disturbed by climatic conditions.
- The amount of certain microorganisms in the soil varies in a significant way that can be modelled by alpha stable random field. The measurement of microorganisms can be taken with a constant error and with some jumps when encountering rocks in the ground.

The proposed estimator depends on a smoothing parameter. Among the perspectives of this work is to find the optimal value of this parameter having the best convergence.

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